

# Reliable Computing I

## Lecture 2: Reliability Metrics

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## Today's Lecture

### ■ Definition, metrics, and terminology

**fault-tol-er-ant** \ˈfɒlt-ˈtäl(-ə)-rənt\  
*adj* : able to function in the  
absence of a major component



## Goals of Fault Tolerant Systems



- How can we deal with problems?
- Option 1: Make problems less likely
  - Tough to do!
  - Testing and design for test (DFT) can help avoid physical defects
  - Careful design reviews can help to avoid design bugs
  - Training and practice can help to avoid operator error
- Option 2: Fail, but don't corrupt anything
  - Example: ATM should shut down instead of passing out money
- Option 3: Transparently tolerate problems
  - Use hardware and/or software to mask fault effects
  - Key: use **redundancy** (a.k.a. spares or backups)
  - Example: having a co-pilot on an airplane

## Reliable Computing System



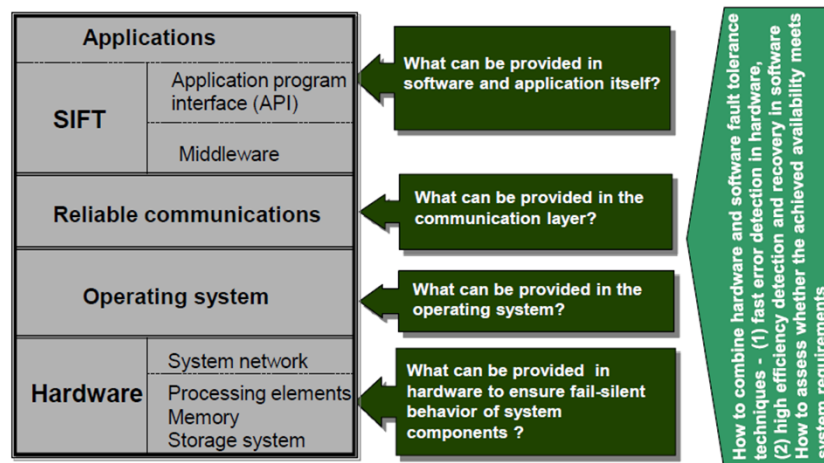
- Correct outputs
  - Desired performance, power consumption
- Changing/varying environmental conditions
  - Power supply, radiation, noise
- Manufacturing process conditions
  - Defects, process variation
- Design errors

## Reliability approaches

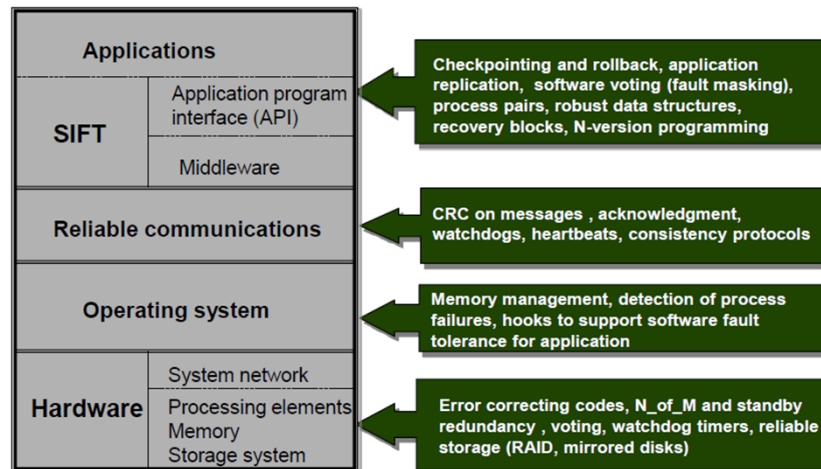


- **Fault avoidance: eliminate problem sources**
  - Remove defects: Testing and debugging
  - Robust design: reduce probability of defects
  - Minimize environmental stress: Radiation shielding etc
  - Impossible to avoid faults completely
    - Occurrence of failures minimized
- **Fault tolerance: add redundancy to mask effect**
  - Failures during system operation
  - Recovery & repair
  - Examples:
    - Error correction coding
    - Backup storage
    - Spare tire

## System View of Dependable Computing



## How do We Achieve the Objectives?



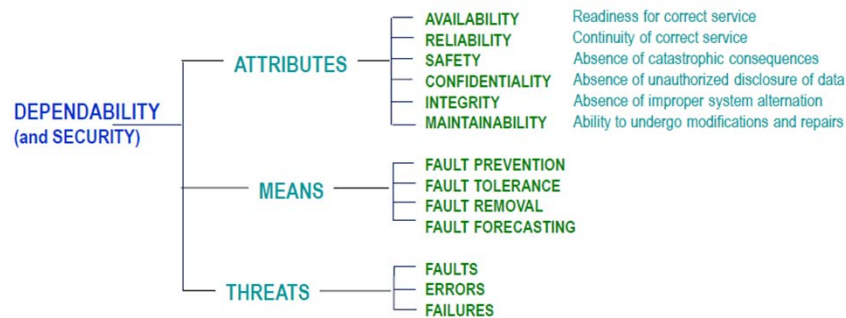
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## Dependable Computing

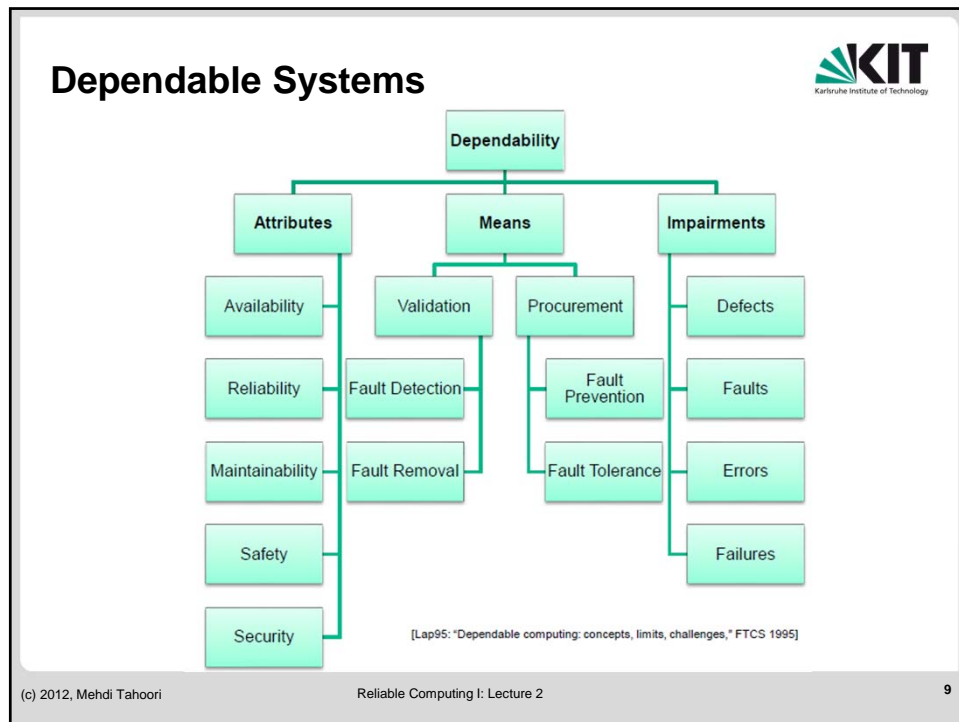
- Original definition of dependability (that stresses the need for justification of trust) states that: *the dependability is the ability to deliver service that can justifiably be trusted*
- The alternate definition (that provides the criterion for deciding if the service is dependable) states that: *the dependability of a system is the ability to avoid service failures that are more frequent and more severe than is acceptable*




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## Intuitive Concepts



- Reliability – continues to work
- Availability – works when I need it
- Safety – does not put me in jeopardy
- Performability - combination of reliability & performance
  - “Graceful degradation”: loss of performance due to minor failures
- Maintainability - ease of repairing a system after failure
- Testability - ease of detecting presence of a fault
- **Survivability** – will the system survive catastrophic events?

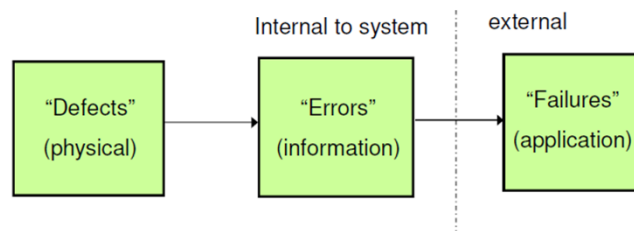
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## Something is wrong...



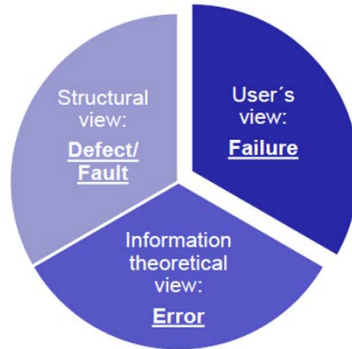
- Defect
  - Distortion of the physical shape
- Fault
  - Logical model of defects
- Error
  - Incorrect signal values/state/information in computation
- Failure
  - Deviation from designed characteristics
  - Observed malfunction during operation
  - Loss of intended function

## Something is wrong...



- Latent fault: which has not yet produced error
  - Faulty component will produce error only when used by a process.
- Latent error: which has not yet produced failure.
  - An infected person may not show symptoms of a disease.

## Something is wrong...



- **Fault:** abstraction of physical defect or bug to structural level
- **Error:** effect of an physical defect, bug
- **Failure:** malfunction of the system, breakdown

## What to do about faults?

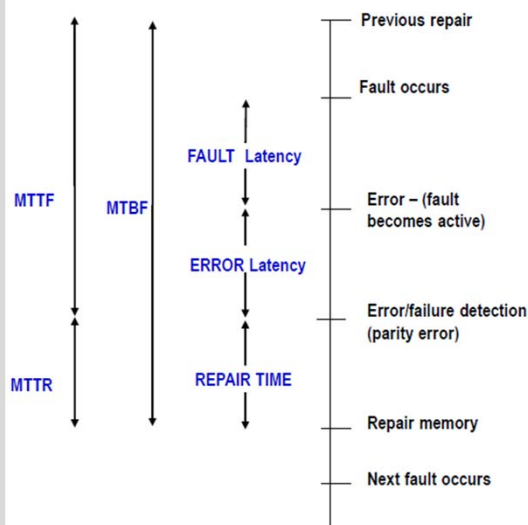
- Finding & identifying faults:
  - **Fault detection:** is a fault there?
  - **Fault location:** where?
  - **Fault diagnosis:** which fault it is?
- Automatic handling of faults
  - **Fault containment:** blocking error flow
    - **Fault masking:** fault has no effect
  - **Fault recovery:** back to correct operation

## System Response to Faults



- Error on output: may be acceptable in non-critical systems if happens only rarely
- **Fault masking**: output correct even when fault from a specific class occurs
  - Critical applications: air/space/manufacturing
- **Fault-secure**: output correct or error indication
  - Retryable: banking, telephony, payroll
- **Fail safe**: output correct or in safe state
  - Flashing red traffic light, disabled ATM

## Fault Cycle & Dependability Measures

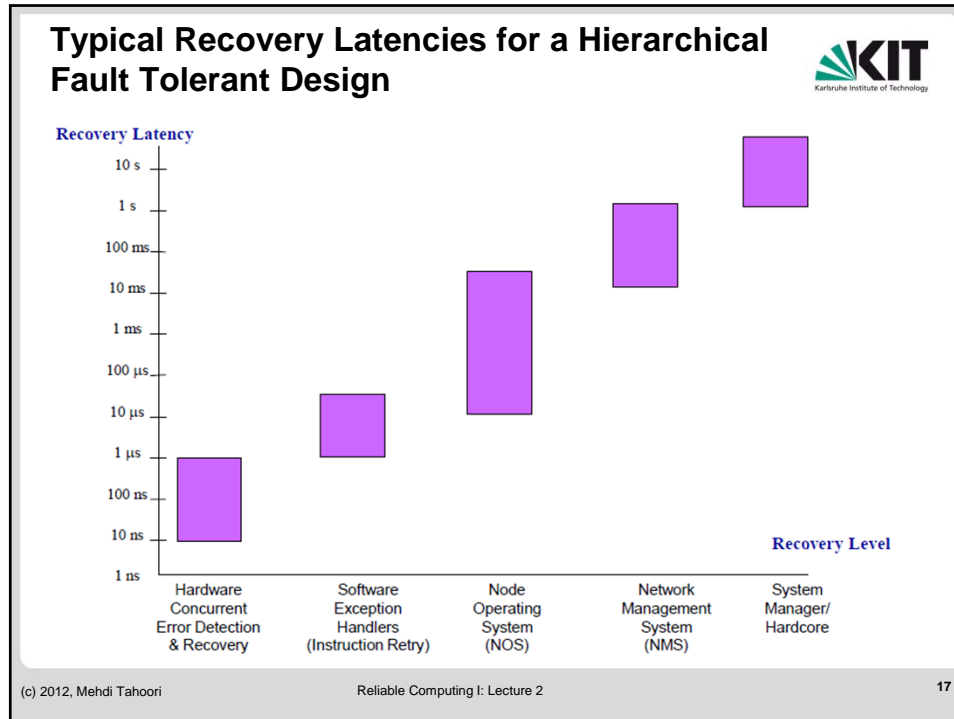


**Reliability:**  
a measure of the continuous delivery of service;  
 $R(t)$  is the probability that the system survives (does not fail) throughout  $[0, t]$ ;  
expected value: **MTTF (Mean Time To Failure)**

**Maintainability:**  
a measure of the service interruption  
 $M(t)$  is the probability that the system will be repaired within a time less than  $t$ ;  
expected value: **MTTR (Mean Time To Repair)**

**Availability:**  
a measure of the service delivery with respect to the alternation of the delivery and interruptions  
 $A(t)$  is the probability that the system delivers a proper (conforming to specification) service at a given time  $t$ .  
expected value:  **$EA = MTTF / (MTTF + MTTR)$**

**Safety:**  
a measure of the time to catastrophic failure  
 $S(t)$  is the probability that no catastrophic failures occur during  $[0, t]$ ;  
expected value: **MTTCF (Mean Time To Catastrophic Failure)**



## First some probabilities...

- For each random variable  $X$ ,
  - cumulative distribution function (CDF):  $F(x) = P(X \leq x)$ 
    - Probability  $P$  that event  $X$  is less than or equal to value of  $x$
  - Probability mass function (PMF):  $F(x) = P(X = x)$
  - Probability density function (PDF):  $f(x) = \frac{dF}{dx}$ 
    - Such that in general  $P(a \leq x \leq b) = \int_a^b f(x) dx$
  - Mean or Expected value:  $E[X] = \int_{-\infty}^{+\infty} xf(x)dx$
  - Variance:  $\sigma_x^2 = E[(x - E[x])^2]$

## Probability of Failure



- Random variable  $T$  is time to the next failure
  - Lifetime of a module (time until it fails)
- $F(t) = \text{Prob} \{T \leq t\}$ 
  - Probability that component will fail before or at time  $t$
- $f(t) = \frac{dF(t)}{dt}$ ,  $\int_0^{\infty} f(t)dt = 1$ ,  $f(t) \geq 0$  (for all  $t \geq 0$ )
  - The momentary rate of probability of failure at time  $t$
- $F$  and  $f$  are related through:
  - $$f(t) = \frac{dF(t)}{dt} \qquad F(t) = \int_0^t f(s)ds$$

## Reliability $R(t)$



- Probability that the system has been operating correctly and continuously from time 0 until time  $t$ , given that it was operating correctly at time 0
  - $R(t) = \text{Prob} \{T > t\} = 1 - F(t)$
- MTTF: Mean Time To Failure
  - Expected value of the lifetime  $T$ 

$$MTTF = E[T] = \int_0^{\infty} t \cdot f(t)dt$$
  - With  $\frac{dR(t)}{dt} = -f(t)$  follows:
 
$$MTTF = -\int_0^{\infty} t \cdot \frac{dR(t)}{dt} \cdot dt = -tR(t) \Big|_0^{\infty} + \int_0^{\infty} R(t)dt = \int_0^{\infty} R(t)dt$$

## Failure Rate $\lambda$



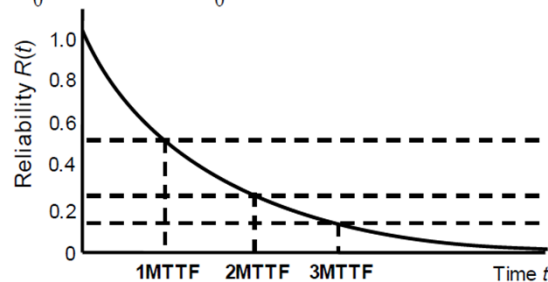
- Number of failures per time unit w.r.t. number of surviving components
  - Also known as hazard function,  $z(t)$
- $\lambda(t) = z(t) = \frac{dF(t)/dt}{(1-F(t))} = \frac{f(t)}{R(t)}$
- A module has a constant failure rate if and only if  $T$  has an exponential distribution
 
$$R(t) = e^{-\lambda t}; F(t) = 1 - e^{-\lambda t}; R(0) = 1$$

$$f(t) = \lambda e^{-\lambda t}$$

## Failure Rate $\lambda(t) = \lambda$



$$MTTF = \int_0^{\infty} t \cdot \lambda e^{-\lambda t} dt = \int_0^{\infty} e^{-\lambda t} dt = \frac{1}{\lambda}$$



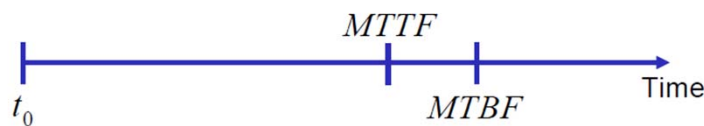
- Reliability  $R(t) = e^{-\lambda t}$

## Availability



- Availability  $A(t)$ 
  - Fraction of time system is operational during the interval  $[0, t]$ 
    - Excludes time for recovery or repair
- MTTR: Mean Time To Repair
- MTBF: Mean Time Between Failures
  - $MTBF = MTTF + MTTR$

$$A = \frac{E[Uptime]}{E[Uptime] + E[Downtime]} = \frac{MTTF}{MTTF + MTTR} = \frac{MTTF}{MTBF}$$



## Other failure distribution models



- Weibull distribution
  - $\alpha$  : shape parameter
    - $\alpha < 1$  : failure rate decreasing with time
    - $\alpha = 1$  : failure rate constant
    - $\alpha > 1$  : failure rate increasing with time
  - $\lambda$  : scale parameter
  - PDF =  $f(t) = \alpha\lambda(\lambda t)^{\alpha-1}e^{-(\lambda t)^\alpha}$
  - CDF =  $F(t) = 1 - e^{-(\lambda t)^\alpha}$
  - Reliability =  $R(t) = e^{-(\lambda t)^\alpha}$

## Other failure distribution models



### ■ Geometric distribution

- Discrete times 0, 1, 2, ...
  - Replacing  $e^{-\lambda}$  by discrete probability  $q$
  - Replacing  $t$  by  $n$
- PMF =  $f(n) = q^n - q^{n+1} = q^n(1 - q)$
- CDF =  $F(n) = 1 - q^n$
- Reliability =  $R(n) = q^n$
- $\mu = \frac{1}{1-q}$  ,  $\sigma = \frac{q^{1/2}}{1-q}$

### ■ Discrete Weibull distribution

## Maintainability



### ■ MTTR may be subdivided as follows

- Time needed to detect a fault and isolate the responsible components (diagnosis)
- Time needed to replace the faulty component
- Time needed to verify that the fault has been removed and the system is fully operational

### ■ Design for maintainability

- System design which supports efficient fault detection, isolation and repair

## Performability



- *Accomplishment levels*  $L_1, L_2, \dots, L_n$  defined in the application context
  - Representing a level of *quality of service* delivered by the application
  - *E.g.*:  $L_i$  indicates  $i$  system crashes during mission time
- Performability is a vector  $(P(L_1), P(L_2), \dots, P(L_n))$ 
  - $P(L_i)$  : Probability that the system performs well enough that the application reaches level  $L_i$