Karlsruhe Institute of Technology

Reliable Computing I

Lecture 5: Reliability Evaluation

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Today's Lecture

- Reliability evaluation
  - Permanent and temporary failures
- Combinatorial modeling
  - Series
  - Parallel
  - Series-parallel
  - Non-series-parallel
  - k-out-of-n
  - TMR vs. Simplex
  - Effects of voter, coverage

Evaluation Criteria

- A method of evaluation is required in order to compare the redundancy techniques and make subsequent design tradeoffs
- Modeling techniques are very vital means for obtaining reasonable predictions for system reliability and availability
  - Combinatorial: series/parallel, K-of-N, nonseries/nonparallel
  - Markov: time invariant, discrete time, continuous time, hybrid
  - Queuing
- Using these techniques probabilistic models of systems can be created and used to evaluate system reliability and/or availability

Basic Reliability Measures

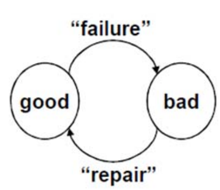
- Reliability: durational (default)
  - R(t)=P{correct operation in duration (0,t)}
- Availability: instantaneous
  - A(t)= P{correct operation at instant t)}
  - Applied in presence of temporary failures
  - A steady-state value is the expected value over a range of time.
- Transaction Reliability: single transaction
  - R_t=P{a transaction is performed correctly}

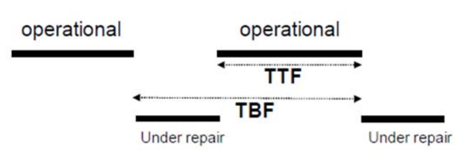
Mean time to …

- Mean Time to Failure (MTTF):
  - expected time the unit will work without a failure.
- Mean time between failures (MTBF):
  - expected time between two successive failures.
    - Applicable when faults are temporary.
    - The time between two successive failures includes repair time and then the time to next failure.
- Mean time to repair (MTTR):
  - expected time during which the unit is non-operational.

Failures with Repair

- Time between failures: time to repair + time to next failure

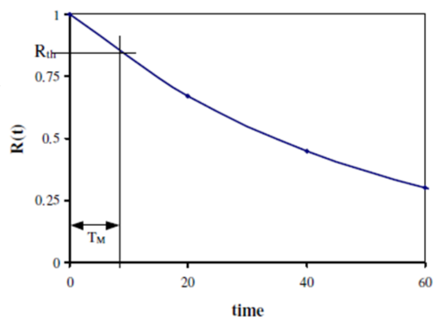




- MTBF = MTTF + MTTR
- MTBF, MTTF are same same when MTTR ≈ 0
- Steady state availability = MTTF / (MTTF+MTTR)

Mission Time (High-Reliability Systems)

- Reliability throughout the mission must remain above a threshold reliability R_{th}.
- Mission time T_M: defined as the duration in which $R(t) \geq R_{th}$.
- R_{th} may be chosen to be perhaps 0.95.
- Mission time is a strict measure, used only for very high reliability missions.



Two Basic cases

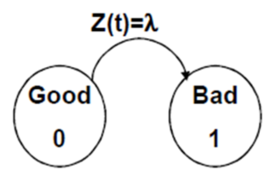
- We next consider two very important basic cases that serve as the basis for time-dependent analysis.

1. Single unit subject to permanent failure
   - We will assume a constant failure rate to evaluate reliability and MTTF.
2. Single unit with temporary failures
   - System has two states Good and Bad, and transitions among them are defined by transition rates.

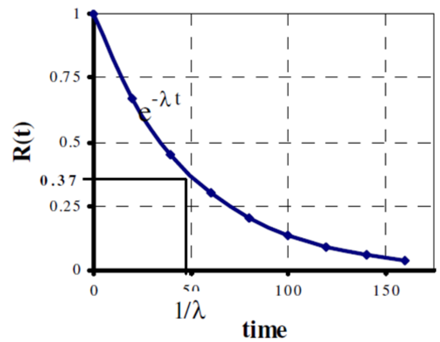
- Both of these are example of Markov processes.

Single Unit with Permanent Failure

- Assumption: constant failure-rate λ
- Reliability = $R(t) = e^{-\lambda t}$
- $MTTF = \int_0^\infty R(t)dt = \int_0^\infty e^{-\lambda t}dt = \frac{1}{\lambda}$

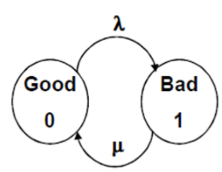


- **Ex 1:** a unit has MTTF =30,000 hrs. Find failure rate.
  $\lambda = 1/30{,}000 = 3.3\times10^{-5}$/hr
- **Ex 2:** Compute mission time T_M if R_{th} =0.95.
  $e^{-\lambda T_M} = 0.95$ $T_M = -\ln(0.95)/\lambda$
  $\approx 0.051/\lambda$
- **Ex 3:** Assume $\lambda = 3.33\times10^{-5}$, and R_{th} =0.95 find T_M.
  Ans: T_M = 1538.8 hrs
  (compare with MTTF =30,000)

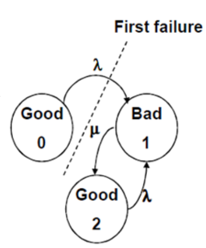


Single Unit: Temporary Failures

- Temporary: intermittent, transient, permanent with repair



- $p_0(t) = p_0(0)e^{-(\lambda+\mu)t} + \frac{\mu}{\lambda+\mu}(1 - e^{-(\lambda+\mu)t})$
- $p_1(t) = 1 - p_0(t)$
- Availability $A(t) = p_0(t)$
- Steady-state availability $(t \to \infty)$ $A(t) = \frac{\mu}{\lambda+\mu}$
- Reliability: R(t) = P{no failure in (0,t)} = $e^{-\lambda t}$
- $MTTF = \frac{1}{\lambda}$
  - Same as permanent failure



Combinatorial Modeling

- System is divided into non-overlapping modules
- Each module is assigned either a probability of working, P_i, or a probability as function of time, $R_i(t)$
- The goal is to derive the probability, P_{sys}, or function $R_{sys}(t)$ of correct system operation
- Assumptions:
  - module failures are independent
  - once a module has failed, it is always assumed to yield incorrect results
  - system is considered failed if it does not satisfy minimal set of functioning modules
  - once system enters a failed state, other failures cannot return system to functional state
- Models typically enumerate all the states of the system that meet or exceed the requirements of correctly functioning system

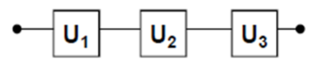
Combinatorial Reliability

- Objective is: Given a
  - systems structure in terms of its units
  - reliability attributes of the units
  - some simplifying assumptions
- We need to evaluate the overall reliability measure.
- There are two extreme cases we will examine first:
  - Series configuration
  - Parallel configuration
  - Other cases involve combinations and other configurations.
- Note that conceptual modeling is applicable to R(t), A(t), $R_t(t)$. A system is either good or bad.

Series configuration

- Assume system has n components, e.g. CPU, memory, disk, terminal
- All components should survive for the system to operate correctly

$$R_S = P\{U_1\, good \cap U_2\, good \cap U_3\, good\}$$
$$= P\{U_1\, g\}P\{U_2\, g\}P\{U_3\, g\}$$
$$= R_1 R_2 R_3$$



- Reliability of the system

$R_{series}(t) = \prod_{i=1}^{n} R_i(t)$ where $R_i(t)$ is the reliability of module i

Series configuration

- For exponential failure rate of each component

$$\text{If } R_i(t) = e^{-\lambda_i t}$$
$$\text{then } R_s(t) = \Pi e^{-\lambda_i t} = e^{-[\lambda_1 + \lambda_2 + \cdots + \lambda_n]t}$$
$$R_{series}(t) = e^{-\sum_{i=1}^{n} \lambda_i t} = e^{-\lambda_{system} t}$$

Where $\lambda_{system} = \sum_{i=1}^{n} \lambda_i$ corresponds to the failure rate of the system

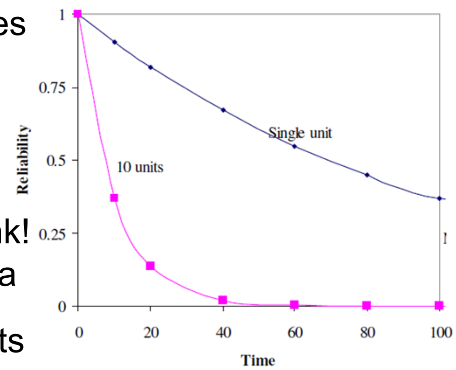
- System failure rate is the sum of individual failure rates:

$$\lambda_S = \lambda_1 + \lambda_2 + \cdots + \lambda_n$$

- Mean time to failure: $MTTF_{series} = \frac{1}{\sum_{i=1}^{n} \lambda_i}$

"A chain is as strong as it's weakest link"?

- Let us see for a 4-unit series system
  - Assume $R_1=R_2=R_3=0.95$, $R_4=0.75$
    - $R_S=0.643$
- Thus a chain is slightly weaker than its weakest link!
- The plot gives reliability of a 10-unit system vs a single system. Each of the 10 units are identical.
  - More units, less reliability

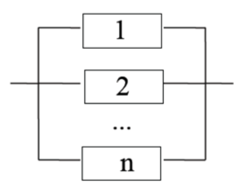


if $X_i \equiv$ lifetime of component i then

$$0 \le E[X] \le \min\{E[X_i]\}$$

Parallel Systems

- Assume system with spares
- As soon as fault occurs a faulty component is replaced by a spare
- Only one component needs to survive for the system to operate correctly
- Prob. of module i to survive = R_i
- Prob. of module i not to survive = $(1 - R_i)$
- Prob. of no modules to survive =
  - $(1 - R_1)(1 - R_2) \dots (1 - R_n)$
- Prob [at least one module survives] =
  - 1 - Prob [none module survives]
- Reliability of the parallel system



$$R_{parallel}(t) = 1.0 - \prod_{i=1}^{n} \left(1.0 - R_i(t)\right)$$

Parallel Systems

$$E(X) = \int_0^{\infty} [1-(1-e^{-\lambda t})^n]dt$$

$$= \ \ldots$$

$$= \frac{1}{\lambda} \sum_{i=1}^{n} \frac{1}{i}$$

$$\approx \frac{\ln(n)}{\lambda}$$

Parallel Configuration: Example

- Problem: Need system reliability $R_s = 1 - \epsilon$
  - How many parallel units are needed
    - If $R_1 = R_2 = \ \ldots = R_m, \ R_m < R_s$
- Solution : $1 - R_s = (1-R_m)^x$

$$\in = (1-R_m)^x$$

$$x = \frac{\ln \in}{\ln(1-R_m)}$$

Assume $R_s = 0.9999$ ($\in = 0.0001$),
$R_m = 0.9$
gives $x = 4$.

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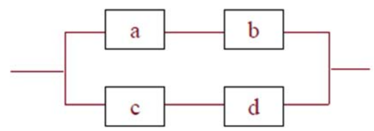
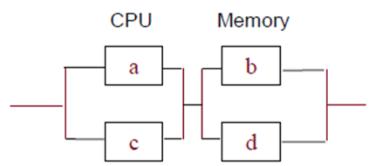
Series-Parallel Systems

- Consider combinations of series and parallel systems
- Example, two CPUs connected to two memories in different ways

$R_{sys} = 1-(1-Ra\,Rb)(1-Rc\,Rd)$

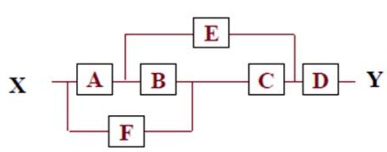


$R_{sys} = (1-(1-Ra)(1-Rc))\,(1-(1-Rb)(1-Rd))$



Non-Series-Parallel-Systems

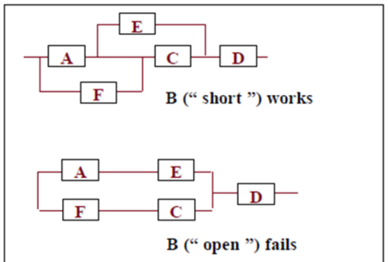
- Often a “success” diagram is used to represent the operational modes of the system



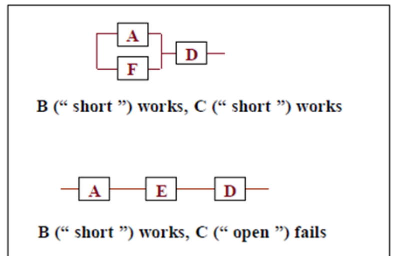
Each path from X to Y represents a configuration that leaves the system operational

- Reliability of the system can be derived by expanding around a single module m
- $R_{sys} = R_m$ P(system works | *m* works) +
  $(1-R_m)$ P(system works | *m* fails)
  - where the notation P(s | m) denotes the conditional probability “s given m has occurred”

Non-Series-Parallel-Systems



Reduced model with B replaced



Reduction with B and C replaced

$R_{sys} = R_B\, P(\text{system works}|\text{B works})$
$+ (1 - R_B)\,\{R_D[1 - (1 - R_A R_E)(1 - R_F R_C)]\}$

$P(\text{system works}|\text{B works}) =$
$R_C\{R_D[1 - (1 - R_A)(1 - R_F)]\}$
$+ (1 - R_C)(R_A R_D R_E)$

Letting $R_A \ldots R_F = R_m$ yields $R_{sys} = R^6_m - 3R^5_m + R^4_m + 2R^3_m$

Non-Series-Parallel-Systems

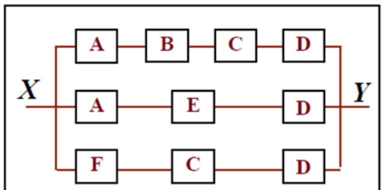
- For complex success diagrams, an upper-limit approximation on R_{sys} can be used
- An upper bound on system reliability is:

  $R_{sys} \le 1 - \prod \left(1 - R_{path\ i}\right)$ R_{path} is the serial reliability of path i

  - The above equation is an upper bound because the paths are not independent.
  - That is, the failure of a single module affects more than one path.

Non-Series-Parallel-Systems

- Example



Reliability block diagram (RBD) of a system

$$R_{sys} \leq 1-(1-R_A R_B R_C R_D)(1-R_A R_E R_D)(1-R_F R_C R_D)$$
$$R_{sys} \leq 2R_m^3 + R_m^4 - R_m^6 - 2R_m^7 + R_m^{10}$$

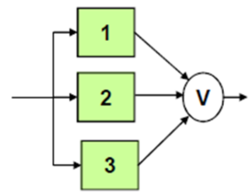
k-out-of-n Systems

- Assumption:
  - we have n identical modules with statistically independent failures.
- k-out-of-n system is operational if
  - k of the n modules are good.
- System reliability then is $R_{k/n} = \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}$
  - Where p is the probability that one unit is good
  - $R_{k/n}$ is the summations of the probabilities of all good combinations
  - $\binom{n}{i} = \frac{n!}{i!(n-i)!}$: choose i good systems out of n

Triple Modular Redundancy

- 2-out-of-3 system

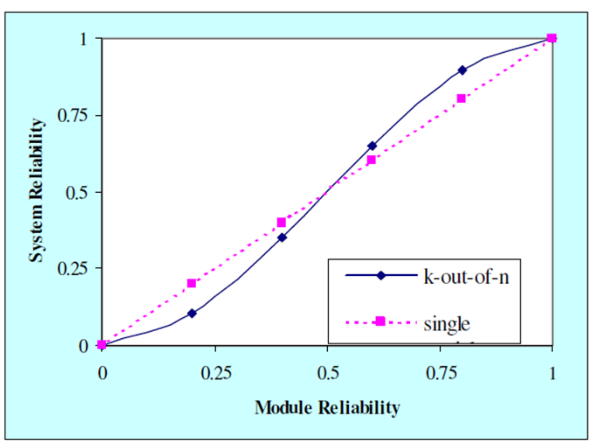
$$R_{TMR} = \sum_{i=2}^{3} \binom{3}{i} R^i (1-R)^{3-i}$$
$$= 3R^2(1-R) + R^3$$
$$= 3R^2 - 2R^3$$



  - Where R is the reliability of a single module.
  - This assumes that the voter is perfect
    - a reasonable assumption if the voter complexity is much less than an individual module.

TMR vs. Simplex

- System reliability vs. module reliability



- What is the conclusion?

TMR vs. Simplex: MTTF

- Compare reliability of simplex and TMR systems

$$R_{simplex}(t) = e^{-\lambda t}$$

$$MTTF_{simplex} = \int e^{-\lambda t} dt = 1/\lambda$$

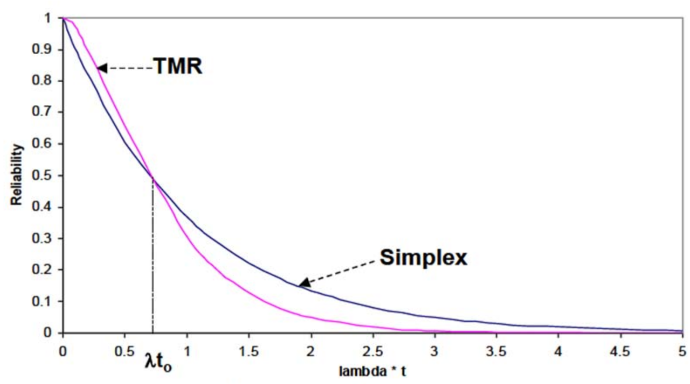
$$R_{TMR}(t) = e^{-3\lambda t} + \binom{3}{2} e^{-2\lambda t}(1 - e^{-\lambda t})$$

$$MTTF = \int_0^\infty R_{TMR}(t)dt = \int_0^\infty (3e^{-2\lambda t} - 2e^{-3\lambda t})dt$$

$$MTTF_{TMR} = \frac{3}{2\lambda} - \frac{2}{3\lambda} = \frac{5}{6\lambda}$$

$$MTTF_{simplex} > MTTF_{TMR}$$

TMR vs. Simplex: MTTF



$$R_{TMR}(t) \geq R(t) \qquad 0 \leq t \leq t_0$$

$$R_{TMR}(t) \leq R(t) \qquad t_0 \leq t < \infty$$

$$\text{where } t_0 = \frac{\ln 2}{\lambda} \approx \frac{0.7}{\lambda}$$

TMR vs. Simplex: Mission Time

- Mission time

$$R_{Th} = 3e^{-2\lambda t_m} - 2e^{-3\lambda t_m}$$

- A numerical solution for t_m can be obtained iteratively
  - $Ex: \lambda = 1/\text{year}, R_{Th} = 0.95$

	$MTTF$	t_m
single	$1yr$	0.05
TMR	0.83	0.145

- Thus TMR mission time is much better.

TMR vs. Simplex: Availability

- Temporary faults: steady state

$$A_{TMR} = 3A^2 - 2A^3, \; A = \frac{\mu}{\lambda + \mu}$$

$$\text{Ex}: \frac{\lambda}{\mu} = 0.01 \Rightarrow A = 0.9901$$

$$\Rightarrow \overline{A} = 0.01$$

$$A_{TMR} = 0.9997 \Rightarrow \overline{A}_{TMR} = 0.0003$$

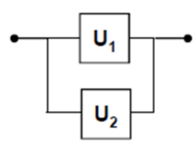
- Thus TMR can greatly reduce down-time in presence of temporary faults

TMR vs. Simplex: Summary

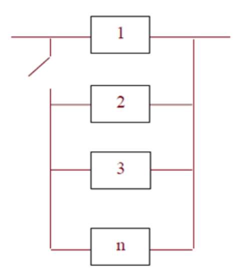
- Instead of MTTF, look at mission time
- Reliability of K-out-of-N systems very high in the beginning
  - spare components tolerate failures
- Reliability sharply falls down at the end
  - system exhausted redundancy, more hardware can possibly fail
- Such systems useful in aircraft control
  - very high reliability, short time
  - 0.99999 over 10 hour period

System with Backup: Effect of Coverage

- Failure detection is not perfect
  - Reconfiguration may not succeed
    - Attach a coverage "c"



$$R_s = P\{U_1\ good\} + P\{U_2\ has\ taken\ over \mid U_1\ failed\}P\{U_1\ failed\}$$
$$= R_1 + R_2 C(1 - R_1)$$
$$where\ \mathrm{C} = \mathrm{P\{failure\ detected\ and\ successful\ switchover\}}$$



- General case, n-1 spares

$$\mathrm{R_s} = \mathrm{R_m} \sum_{i=0}^{n-1} C^i (1 - \mathrm{R_m})^i$$

System with Backup: Effect of Coverage

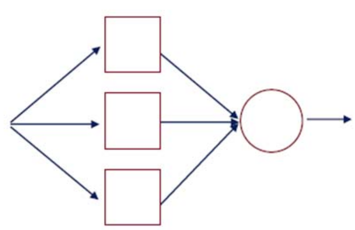
- If coverage is 100%, then given low module reliability, can increase system reliability arbitrarily
  - With low coverage, reliability saturates

	Rm = 0.9	Rm = 0.7	Rm = 0.5
C=0.99, n=2	0.989	0.908	0.748
C=0.99, n=4	0.999	0.988	0.931
C=0.99, n=inf	0.999	0.996	0.990
C= 0.8 , n=2	0.972	0.868	0.700
C= 0.8 , n=4	0.978	0.918	0.812
C=0.8, n=inf	0.978	0.921	0.833

Effect of Voter

- Previous expression for reliability assumed voter 100% reliable
- Assume voter reliability R_v

$$R_{TMRV} = R_V (R_m^3 + \binom{3}{2} R_m^2 (1 - R_m))$$



TMR+Spares

- TMR core, n-3 spares (assume same failure rate)
- System failure when all but one modules have failed.
  - If we start with 3 in the core and 2 spares, the sequence is:
  - 3+2 → 3+1 → 3+0 → 2+0 → failure
- Reliability of the system then is
  $R_s=R_{sw}[1-nR(1-R)^{n-1}-(1-R)^n]$
  - Where R is reliability of a single module and R_{sw} is the reliability of the switching circuit overhead.
  - R_{sw} should depend on total number of modules n, and relative complexity of the switching logic.
- Let us assume that $R_{sw}=(R^a)^n$,
  - where a is measure of relative complexity, generally a <<1
- $R_s=R^{an}[1-nR(1-R)^{n-1}-(1-R)^n]$