Karlsruhe Institute of Technology

Reliable Computing I

Lecture 7: Information Redundancy-2

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Today's Lecture

- Codes for storage and communication
  - Cyclic codes
  - Reed-Solomon codes
- Arithmetic codes
- Self-checking logic

Codes for Storage and Communication

- Cyclic codes are parity check codes with additional property that cyclic shift of codeword is also a codeword
  - if (Cn-1, Cn-2 ... C1, C0) is a codeword, (Cn-2, Cn-3, ... C0, Cn-1) is also a codeword
- Cyclic codes are used in
  - sequential storage devices, e.g. tapes, disks, and data links
  - communication applications
- An (n,k) cyclic code can detect single bit errors, multiple adjacent bit errors affecting fewer than (n-k) bits, and burst transient errors
- Cyclic codes require less hardware, in form of linear feedback shift registers
  - parity check codes require complex encoding, decoding circuit using arrays of EX-OR gates, AND gates, etc.

Cyclic Code and Polynomials

- Cyclic codes depend on the representation of data by a polynomial
- If $(C_{n-1}, C_{n-2} \ldots C_1, C_0)$ is a codeword, its polynomial representation is $C(x)= C_{n-1}\, x^{n-1} + C_{n-2}x^{n-2} + \ldots C_1x + C_0$
- Cyclic codes are characterized by their generator polynomial $g(x)$
- g(x) is a polynomial of degree (n-k) for an (n,k) code, with a unity coefficient in (n-k) term
- g(x) is a factor of x^n-1, i.e., it divides it with zero remainder
  - if a polynomial with degree n-k divides x^n-1, then g(x) generates a cyclic code
- Example: for (7,4) code, $g(x) = x^3 + x + 1$

Cyclic Redundancy Check (CRC)

- Considers dataword and codeword to be polynomials
  - E.g., $i_0, i_1, i_2, \ldots, i_{n-1} \rightarrow i_0 + i_1X + i_2X^2 + \ldots + i_{n-1}X^{n-1}$
- Codeword = Dataword * Generator
  - C(X) = D(X) * g(X)
  - g(X) is pre-defined CRC polynomial
    - depends on particular code
  - Additions performed during multiplication are mod2
    - 0+0 = 0, 0+1 = 1+0 = 1, 1+1 = 0
- At receiver, divide n-bit codeword by CRC polynomial
  - D(X) = C(X) / g(X)
- If remainder is non-zero, we've detected an error

Basic Operations on Polynomials

- Can multiply or divide one polynomial by another, follow modulo 2 arithmetic, coefficients are 1 or 0, and addition and subtraction are same

Multiplication

$(x^4 + x^3 + x^2 + 1)(x^3 + x)$

$$\begin{array}{r} x^7 + x^6 + x^5 + x^3 \\ + x^5 + x^4 + x^3 + x \\ \hline = x^7 + x^6 + x^4 + x \end{array}$$

Division

x Quotient

$x^4 + x^3 + x^2 + 1$) $x^5 + x^4$

$$\begin{array}{r} x^5 + x^4 \\ x^5 + x^4 + x^3 + x \\ \hline x^3 + x \end{array}$$

Remainder

Cyclic Code - Example

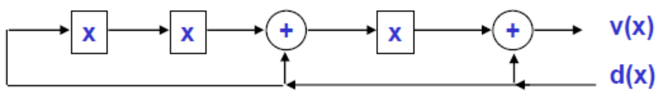
- Consider generator polynomial $g(x) = x^3 + x + 1$ for (7,4) code
- Can verify g(x) divides $x^7 - 1$
- Given data word (1111), generate codeword
  - $d(x) = x^3 + x^2 + x + 1$
- Then $c(x) = g(x)d(x) = (x^3 + x^2 + x + 1)(x^3 + x + 1) = x^6 + x^5 + x^3 + 1$
- Hence code word is (1101001)

CRC Properties and Varieties

- An n-bit CRC check can detect all errors of less than n bits and all but 1 in 2^n multi-bit errors
- Examples:
  - CRC-12: $g(X) = X^{12}+X^{11}+X^3+X^2+X+1$
  - CRC-16: $g(X) = X^{16}+X^{15}+X^2+1$
- Ethernet uses CRC-32
  - More bits $\rightarrow$ better error detection capability

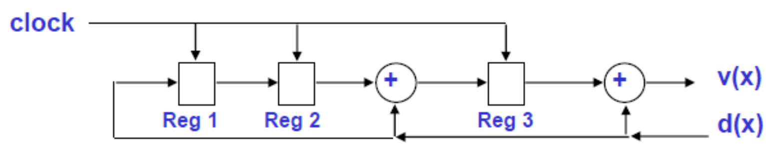
Circuit to Generate Cyclic Code

- Consider blocks labeled X as multipliers, and addition elements as modulo 2



$g(x)=x^3 + x + 1$

- Another representation is to replace multipliers by storage elements, adders by EX-OR gates

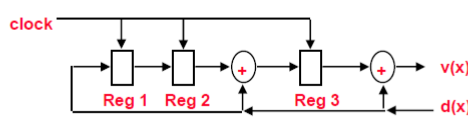


Generation of Code Words

Cyclic codes for 4-bit information words.

Information (d_0, d_1, d_2, d_3)	Code $(v_0,v_1,v_2,v_3,v_4,v_5,v_6)$
0000	0000000
0001	0001101
0010	0011010
0011	0010111
0100	0110100
0101	0111001
0110	0101110
0111	0100011
1000	1101000
1001	1100101
1010	1110010
1011	1111111
1100	1011100
1101	1010001
1110	1000110
1111	1001011

Data polynomial $= d_0 + d_1x + d_2x^2 + d_3x^3$
Generator polynomial $= 1 + x + x^3$
Code polynomial $= v_0 + v_1x + v_2x^2 + v_3x^3 + v_4x^4 + v_5x^5 + v_6x^6$



The encoding process

Clock period	Register values 1	2	3	D(x)	V(x)
0	0	0	0	1	1
1	1	0	1	1	0
2	1	1	1	0	1
3	0	1	1	1	0
4	1	0	0	0	0
5	0	1	0	0	0
6	0	0	1	0	1
7	0	0	0		

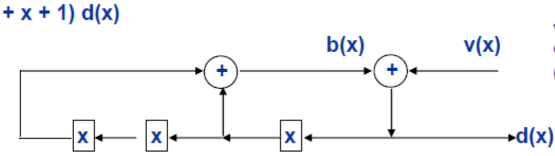
Decoding of Cyclic Codes

- Determine if code word $(r_{n-1}, r_{n-2}, \ldots\ldots, r_1, r_0)$ is valid
- Code polynomial $r(x) = r_{n-1}\, x^{n-1} + r_{n-2}x^{n-2} + \ldots\, r_1x + r_0$
- If $r(x)$ is a valid code polynomial, it should be a multiple generator polynomial $g(x)$
- $r(x) = d(x)\, g(x) + s(x)$, where **$s(x)$ the syndrome polynomial** should be zero
- Hence, divide $r(x)$ by $g(x)$ and check the remainder whether equal to 0

Circuits for Decoding

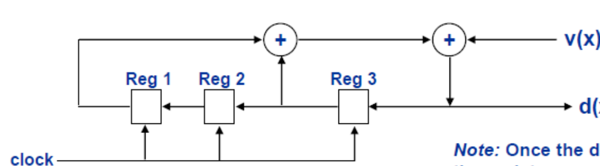
$b(x) = (x^3 + x)\, d(x)$
$v(x) + b(x) = d(x)$
$v(x) = (x^3 + x + 1)\, d(x)$

Hence, $d(x) = v(x) / (x^3 + x + 1)$



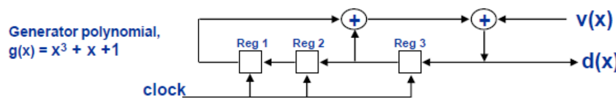
$v(x) = d(x) - b(x) =$
$d(x) - (x^3 + x)\, d(x) =$
$(x^3 + x + 1)dx$

Another representation is to replace multipliers by storage elements and adders by EX-OR gates



Note: Once the division is completed, the registers contain the value of the syndrome (remainder)

Example Decoding



The decoding process with correct information

Clock period	Register values 1	2	3	V(x)	B(x)	D(x)
0	0	0	0	1	0	1
1	0	0	1	0	1	1
2	0	1	1	1	1	0
3	1	1	0	0	1	1
4	1	0	1	0	0	0
5	0	1	0	0	0	0
6	1	0	0	1	1	0
7	0	0	0			

Syndrome

Code word

Original Information

The decoding process with erroneous information

Clock period	Register values 1	2	3	V(x)	B(x)	D(x)
0	0	0	0	1	0	1
1	0	0	1	0	1	1
2	0	1	1	1	1	0
3	1	1	0	**1**	1	0
4	1	0	0	0	1	1
5	0	0	1	0	1	1
6	0	1	1	1	1	0
7	1	1	0			

Nonzero Syndrome

Received word

Systematic Cyclic Codes

- Previous cyclic codes were not systematic, i.e. data not part of code word
- To generate (n,k) systematic cyclic code, do the following:
  - Multiply d(x) by x^{n-k}, this is accomplished by shifting d(x) n-k bits
  - The code polynomial is $c(x) = r(x) + x^{n-k} d(x)$
  - Hence $x^{n-k} d(x) + r(x) = g(x)q(x)$, which is code word c(x) since it is a multiple of g(x)

Example of Systematic Cyclic Code

- Generator polynomial $g(x) = x^4 + x^3 + x^2 + 1$ of (7,3) code
- Data is 3 bits, n-k = 4 bits

Systematic (7, 3) Cyclic Code Generated by $G(x) = x^4 + x^3 + x^2 + 1$

Message Bits $m_2 m_1 m_0$	$x^4M(x)$	$C(x) = \mathrm{Rem}[x^4M(x) \div G(x)]$	Code Word $x^4M(x) - C(x)$ $v_6v_5v_4v_3v_2v_1v_0$
000	0	0	0000000
001	x^4	x^3+x^2+1	0011101
010	x^5	x^2+x+1	0100111
011	x^5+x^4	x^3+x	0111010
100	x^6	x^3+x^2+x	1001110
101	x^6+x^4	$x+1$	1010011
110	x^6+x^5	x^3+1	1101001
111	$x^6+x^5+x^4$	x^2	1110100

$d(x) \times^{n-k}$

Reed-Solomon Codes

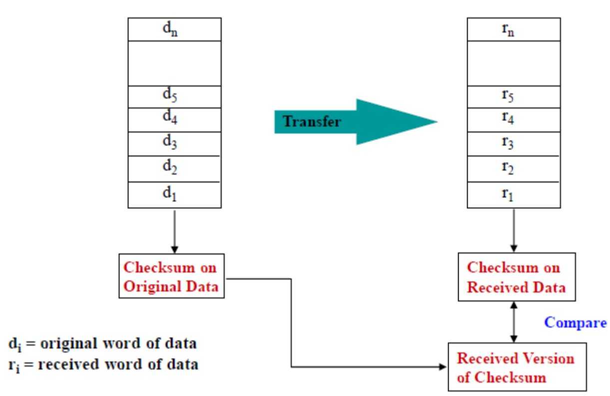
- Popular ECC for CDs, DVDs, wireless communications, etc.
- k data symbols, each of which is s bits
- r parity symbols, each of which is also s bits
- Can correct up to r/2 symbols that contain errors
  - Or can correct up to r symbol erasures
  - Erasure = error in a known symbol
- Denoted by RS(n,k)
- Common example: RS(255, 223) with s=8
  - n = 255 → 255 codeword bytes
  - k = 223 → 223 dataword bytes
  - r = 32 → can correct errors in ≤ 16 bytes

Reed-Solomon Codes,

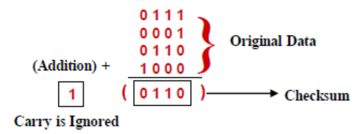
- There exist many flavors of RS codes, each of which is tailored to specific purpose
  - Cross-Interleaved Reed-Solomon Coding (CIRC) used in CDs can correct error burst of up to 4000 bits!
  - 4000 bits is roughly equivalent to 2.5mm on the CD surface
- RS codes are best for bursty error model
  - Just as good at handling 1 error in symbol or s errors in symbol
- Codewords created by multiplying datawords with generator polynomial (like CRC)

Checksum Codes - Basic Concepts

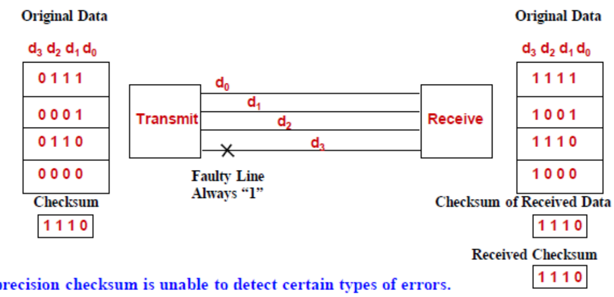
- The checksum is appended to block data when such blocks are transferred



Single Precision Checksums



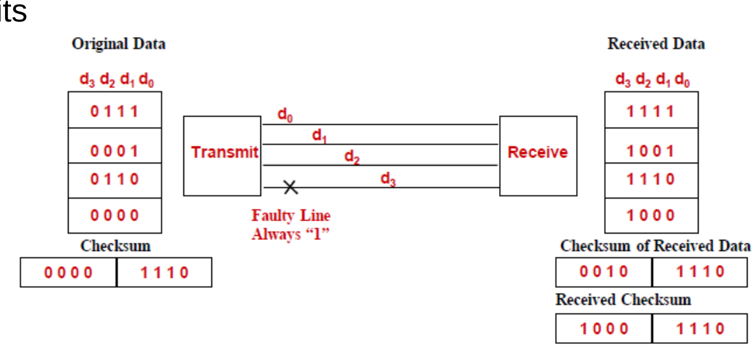
A single-precision checksum is formed by adding the data words and ignoring any overflow



The single-precision checksum is unable to detect certain types of errors. The received checksum and the checksum of the received data are equal, so no error is detected.

Double Precision Checksums

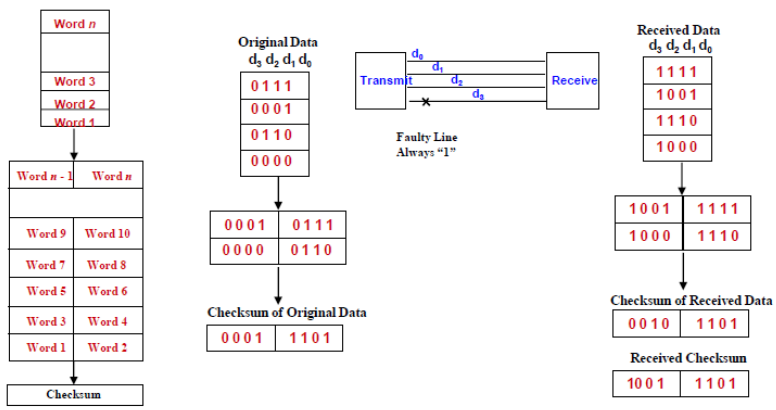
- Compute 2n-bit checksum for a block of n-bit words
- Overflow is still a concern, but it is now overflow from a 2n-bits



The received checksum and the checksum of the received data are not equal, so the error is detected

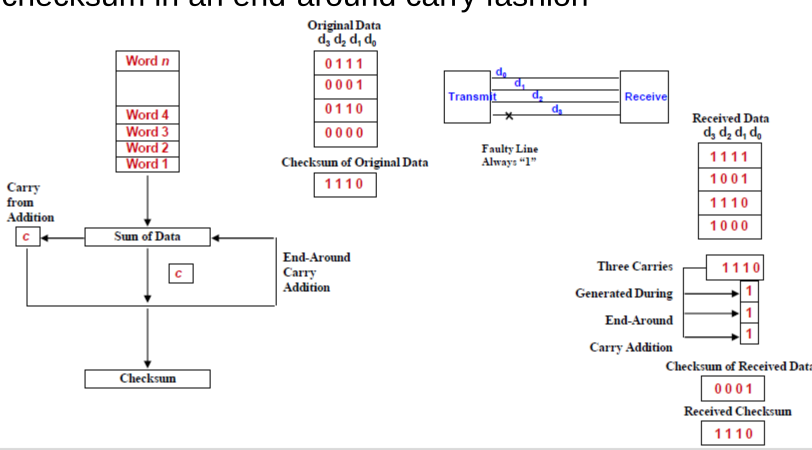
Honeywell Checksums

- Concatenate consecutive words to form double words to create k/2 words of 2n bits; checksum formed over newly structured data



Residue Checksums

- The same concept as the single-precision checksum except that the carry bit is not ignored and is added to checksum in an end-around carry fashion

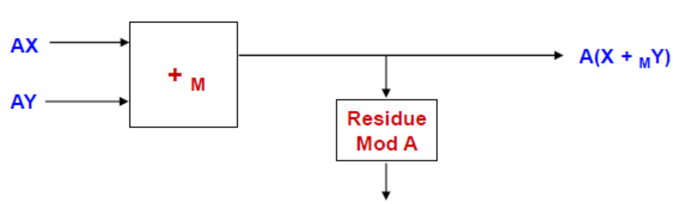


Arithmetic Codes

- Useful to check arithmetic operations
- Parity codes are not preserved under addition, subtraction
- Arithmetic codes can be
  - Separate: check symbols disjoint from data symbols
  - Non-separate: combined check and data
- Several Arithmetic codes
  - *AN codes, Residue codes, Bi-residue codes*
- Arithmetic codes have been used in STAR fault tolerant computer for space applications

AN codes

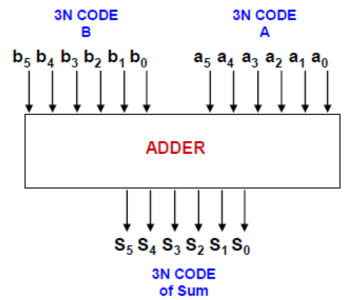
- Data X is multiplied by check base A to form A.X
- Addition of code words performed modulo M where A divides M
- $A(X +_M Y) = AX +_M AY$
- Check operation by dividing the result by A
- If result = 0, no error, else error



Example of 3N Code

Resulting 3N code words for a 4-bit information words

Original Information	3N code word
0000	000000
0001	000011
0010	000110
0011	001001
0100	001100
0101	001111
0110	010010
0111	010101
1000	011000
1001	011011
1010	011110
1011	100001
1100	100100
1101	100111
1110	101010
1111	101101



Normal Operation

A = 0 1 0 0 1 0 (3N Code of 6)
+ B = 0 0 0 0 1 1 (3N Code of 1)
S = 0 1 0 1 0 1 (3N Code of 7)

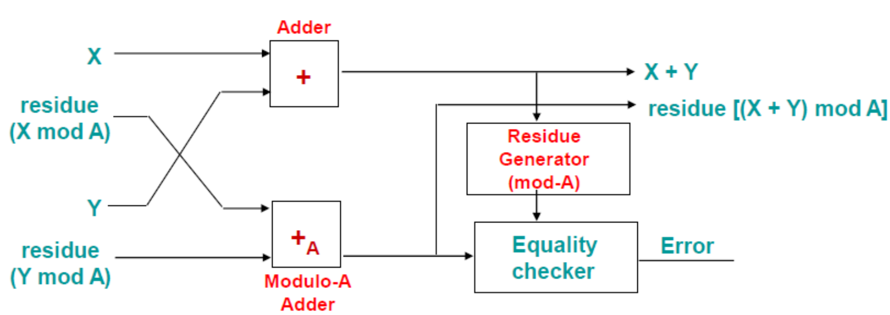
If S_1 is always "1"

A = 0 1 0 0 1 0 (3N Code of 6)
B = 0 0 0 0 1 1 (3N Code of 1)
S = 0 1 0 1 1 1 (Not a valid 3N Code)

Illustration of the error detection capabilities of the 3N arithmetic code. The presence of the fault results in the sum being an invalid 3N code.

Residue Codes

- Separate code (X, X Mod A)
- Created by appending the residue of a number to that number



Berger Codes

- Used in Control units as systematic codes
- The k check bits are the binary encoding of the number of zeros in the d-bit dataword
  - Berger codes are formed by appending $k = \lceil \log_2 (d+1) \rceil$ check bits and n = d + k
- Example:
  - X=1 0 0 1 0 0 0 1 => $k = \lceil \log_2 (8+1) \rceil = 4$
  - the number of 1s in this data is 3 (0011)
  - the complement of (0011) is (1100)
  - the resulting code word is: 1001 0001 1100

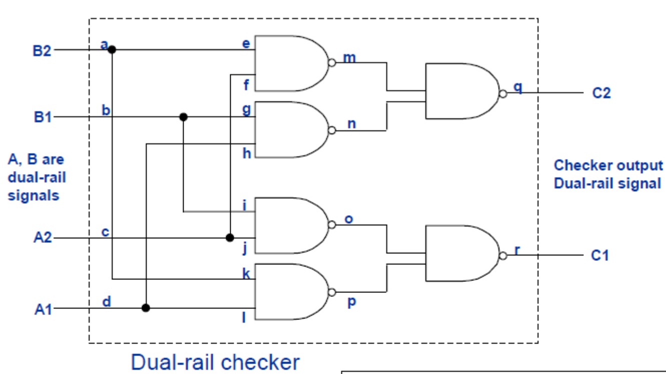
Berger Codes

- Can detect all single-bit errors and all unidirectional multi-bit errors
  - Unidirectional: all bit errors are either from 0→1 or from 1 → 0
- Good for detecting coupling faults
  - Change in one bit erroneously causes change(s) in other bit(s)
  - Models short circuits (including bridging faults)

Self-Checking Circuits

- What properties/invariants can we build into circuits such that codeword inputs do not lead to codeword outputs in the presence of faults?
- Self-testing circuit
  - for every fault from a prescribed set there exists at least one valid input code word that will produce an invalid output code word when a single fault is present in the circuit
- Fault secure circuit
  - any single fault from a prescribed set results in the circuit either producing the correct code word or producing a non-code word, for any valid input code word
- **Totally self-checking circuit** (TSC)
  - the circuit is both fault secure and self-testing
  - all single faults are detectable by at least one valid code word input, and when a given input combination does not detect the fault, the output is the correct code word output

Circuit of Basic TSC Comparison Element



1. Dual-rail signal coded so two bits are complementary.
2. Comparison element checks for the equality of the two dual-rail signals at its inputs.
3. Outputs a dual-rail signal 01 or 10 if both inputs are equal and properly coded; otherwise, outputs 00 or 11.

Implementing EDC/ECC in Hardware

- Where does EDC/ECC get used?
  - Disk, CD-ROM
  - Memory (DRAM, SRAM)
  - Buses
  - Network
- Tradeoff between EDC and ECC
- ECC: Forward error recovery
  - Often on critical path, so can slow down even fault-free system
- EDC: Backward error recovery
  - Detecting error leads to recovery (can be slow)
- So would you use ECC or EDC in your L1 cache?
  - How about in DRAM?