


Testing Digital Systems I

Lecture 3: Quality Models and Yield Analysis

Instructor: M. Tahoori

Copyright 2011, M. Tahoori TDS I: Lecture 3 1



VLSI Chip Yield

- A manufacturing defect is a finite chip area with electrically malfunctioning circuitry caused by errors in the fabrication process.
- A chip with no manufacturing defect is called a good chip.
- Fraction (or percentage) of good chips produced in a manufacturing process is called the *yield*. Yield is denoted by symbol Y .
- Cost of a chip:
$$\frac{\text{Cost of fabricating and testing a wafer}}{\text{Yield} \times \text{Number of chip sites on the wafer}}$$

Copyright 2011, M. Tahoori TDS I: Lecture 3 2



Definitions

- Quality Level, QL
 - Fraction of Parts Passing Test that are Good
- Defect Level, $DL = 1 - QL$
 - Fraction of Parts Passing Test that are ~~Good~~ BAD
 - Measured in DPM, Defects per Million
 - Typical Claim is Less than 200 DPM (0.02 %)
- Yield, Y
 - Fraction of Manufactured Parts that are Good
 - Typically 10 to 90 %
- Reject Ratio
 - Fraction of Manufactured Parts that Fail Test
 - Used to Estimate Yield

Copyright 2011, M. Tahoori

TDS I: Lecture 3

3



Determination of DL

- From field return data:
 - Chips failing in the field are returned to the manufacturer.
 - The number of returned chips normalized to one million chips shipped is the DL.
- From test data:
 - Fault coverage of tests and chip fallout rate are analyzed.
 - A modified yield model is fitted to the fallout data to estimate the DL.

Copyright 2011, M. Tahoori

TDS I: Lecture 3

4



Test Thoroughness

- Measured by
 - Test transparency, TT
 - Fraction of Defects NOT Detected by Test
 - Estimated by FAULTS Missed by Test
 - Faults are Logical Models of Defects
- Required Test Transparency
 - Depends on YIELD and Acceptable Quality Level

Copyright 2011, M. Tahoori

TDS I: Lecture 3

5



Estimating Board Quality Level

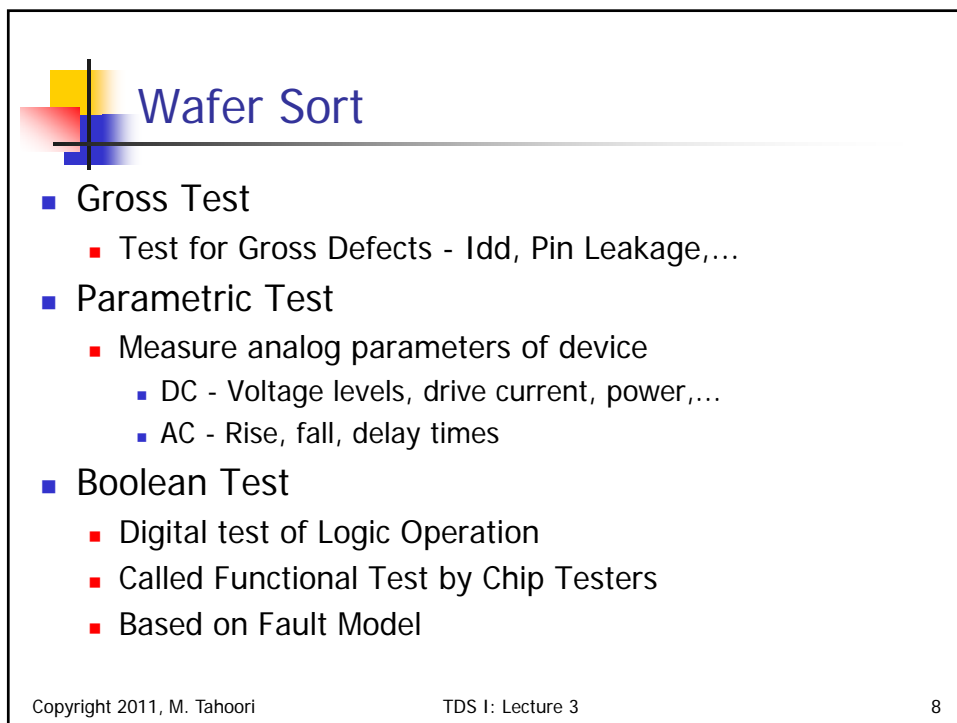
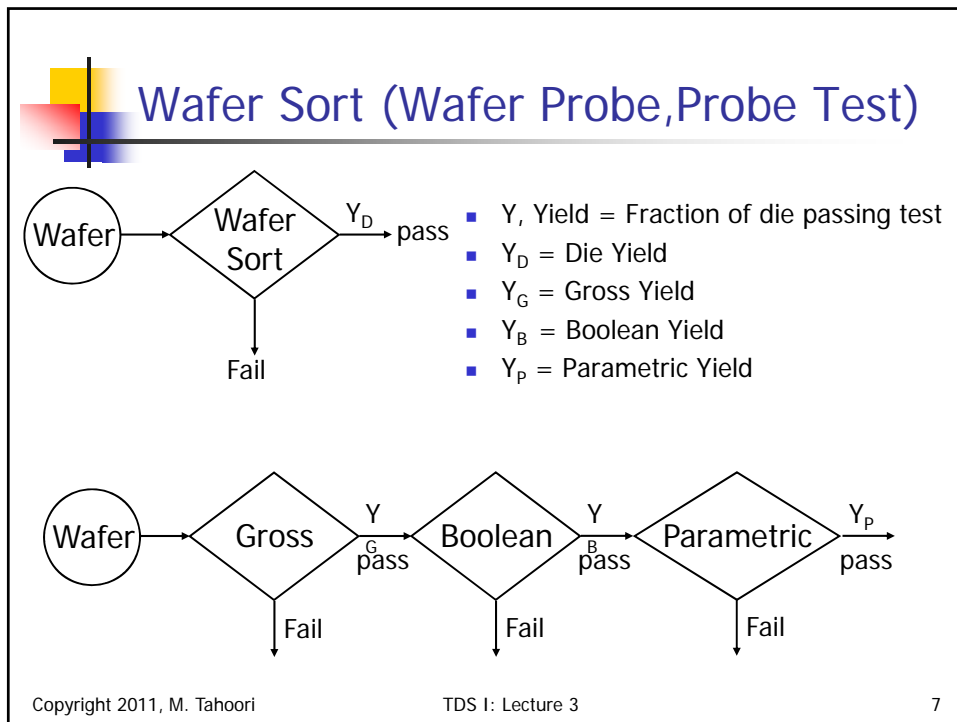
- N Components per Board
- Component Defects are Identical and Independent
- Each Component has Probability, q , of being Defective
- Probability that Board has no Defective Component is:
 - $P = (1 - q)^N$

N	DL	q	P %	1-P
40	10,000 DPM	10^{-2} or 1 %	66.9	33.1
200	10,000 DPM	10^{-2} or 1 %	13.4	86.6
40	1000 DPM	10^{-3} or 0.1 %	96	4
200	1000 DPM	10^{-3} or 0.1 %	82	18
40	100 DPM	10^{-4} or 0.01 %	99.6	0.4
200	100 DPM	10^{-4} or 0.01 %	98	2

Copyright 2011, M. Tahoori

TDS I: Lecture 3

6



Wafer Sort Definitions

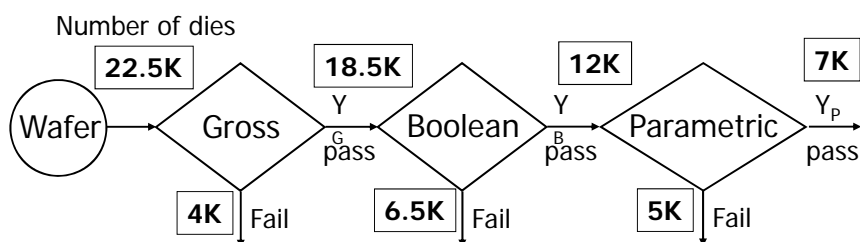
- Die yield, Y_D ,
 - Fraction of Die that are defect-free
 - *ESTIMATED* by Fraction of Die that pass Wafer Sort
- Boolean (functional) defect
 - Defect that Changes Function Realized by Die
- Boolean (functional) Yield, Y_B ,
 - Fraction of Die that are free of Boolean defects
 - *ESTIMATED* by Fraction of Die that Pass Boolean Test
- Boolean (functional) test
 - Test for Boolean (functional) Defects

Copyright 2011, M. Tahoori

TDS I: Lecture 3

9

Motorola6802 Wafer Sort Experiment



- Single Stuck-Fault Coverage = 99.9 %
- $Y_B = 12 / 18.5 = 65.167 \%$
- $Y_G = 18.5 / 22.5 = 82.2 \%$
- $Y_P = 7 / 12.00 = 58 \%$

Copyright 2011, M. Tahoori

TDS I: Lecture 3

10



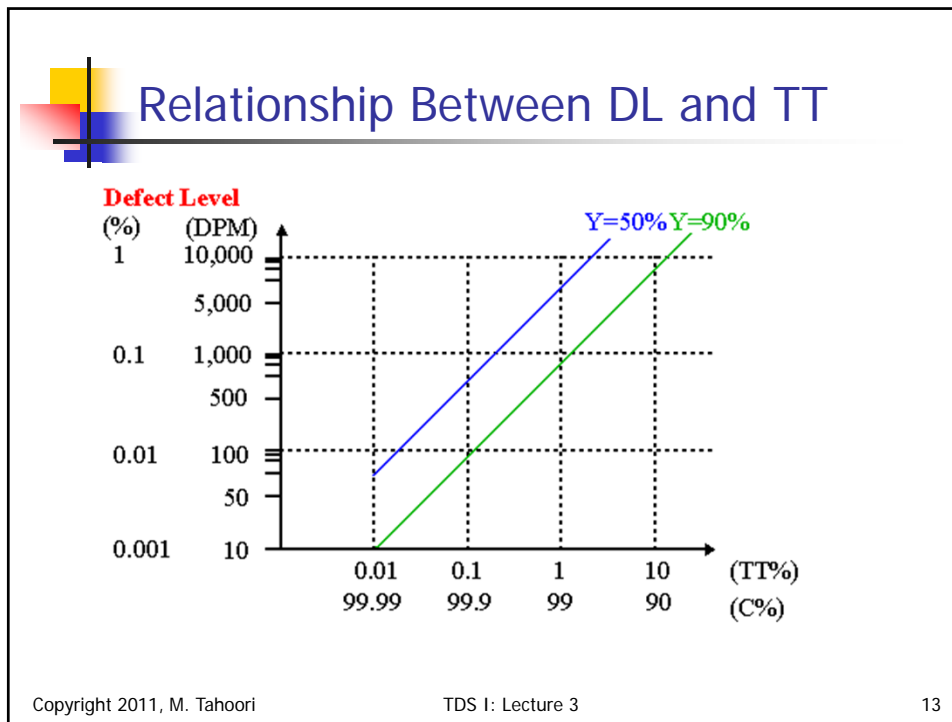
What If

- Single stuck-fault coverage were 96.6 % ?
 - Theory Predicts:
 - $DL = 14,454$ DPM or 174 Bad Die Pass Boolean Test
 - Experiment shows:
 - $DL = 8,471$ DPM or 103 Bad Die Pass Boolean Test



Quality Level Dependence On Y & TT

- Theorem:
 - The Boolean Quality Level achieved by a test with Boolean Test Transparency, TT, for a process of Boolean Yield, Y, is given by:
 - $QL = Y^{TT}$
- Corollary:
 - For Tests that Result in Defect Levels, DL, less than 1000 DPM this can be simplified to:
 - $DL = (-\ln Y) TT$
 - And further simplified for $Y \geq 90\%$ to:
 - $DL = (1 - Y) TT$



Derivation Of Theorem (1)

- n Possible Point Defects on Chip
- **Assumption:** defects are independent and equally distributed
- m of the n Defects Detected by Test Set, $(n - m)$ Not Detected
- TT is $(n - m)/n = 1 - (m / n)$
- p is Probability of a Defect Occurring
- A is Event that Die has no Defects
 - Yield $Y = P[A] = (1 - p)^n$
- B is event that die passes test, none of the m defects on die
 - $P[B] = (1 - p)^m$
 - if a chip is free of defects, it is free of m tested defects
 - $P[AB] = P[A] = (1 - p)^n$

Copyright 2011, M. Tahoori TDS I: Lecture 3 14



Derivation Of Theorem (2)

- QL is given by the probability that a chip is free of all n defects when it is known that it is free of any of the m defects detected by the test process
- $QL = \frac{\text{Number of defect-free parts that pass the test}}{\text{Total number of parts that pass the test}}$
- $QL = P[A|B] = P[AB]/P[B] = P[A]/P[B]$
- $= (1 - p)^{(n - m)} = (1 - p)^{n(1 - m/n)} = Y^{TT}$



Simplifications (1)

- DL less than 1,000 DPM
 - $DL < 10^{-3} \Rightarrow QL = 1 - DL = Y^{TT} > 0.999$
 - $|TT \ln Y| < |\ln(0.999)| = 10^{-3}$
 - Series expansion of $QL = Y^{TT}$
 - $QL = Y^{TT} = 1 + TT \ln Y + (TT \ln Y)^2 / 2 + (TT \ln Y)^3 / 3! + \dots$
 - Since $|TT \ln Y| < 10^{-3}$
 - $|(TT \ln Y)^2 / 2 + (TT \ln Y)^3 / 3! + \dots| < 10^{-6}$
 - $QL \approx 1 + TT \ln Y$
 - $DL \approx TT (-\ln Y)$
 - $TT \approx -DL / (\ln Y)$



Simplifications (2)

- DL less than 1,000 DPM and $Y = 90\%$
 - $\ln Y = (Y - 1) - (Y - 1)^2 / 2 + (Y - 1)^3 / 3 - \dots$
 - $-\ln Y = (1 - Y) + (1 - Y)^2 / 2 + (1 - Y)^3 / 3 + \dots$
- $Y = 90\% \Rightarrow$
 - $(1 - Y) < 0.1$ and
 - $|1 / 2 (1 - Y)^2 + 1 / 3 (1 - Y)^3 - \dots| < 10^{-2}$
 - $(1 - Y) < (-\ln Y) < (1 - Y) + 10^{-2}$

$$DL \approx TT (-\ln Y) \approx TT (1 - Y)$$

Copyright 2011, M. Tahoori

TDS I: Lecture 3

17



Example


- Required TT and coverage (C) for DL=200 DPM

Y%	10	50	75	90	95	99
$-\ln Y$	2.3	0.69	0.288	0.105	0.05	0.01
$1/(-\ln Y)$	0.434	1.44	3.48	9.49	19.5	99.5
TT%	0.008	0.03	0.07	0.2	0.4	2
C%	99.992	99.97	99.93	99.8	99.6	98

Copyright 2011, M. Tahoori


TDS I: Lecture 3

18



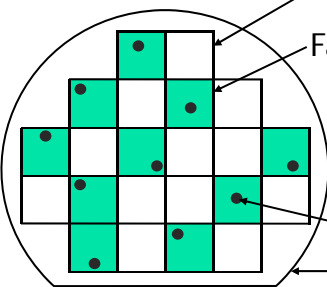
Clustered Defects

Copyright 2011, M. Tahoori TDS I: Lecture 3 19



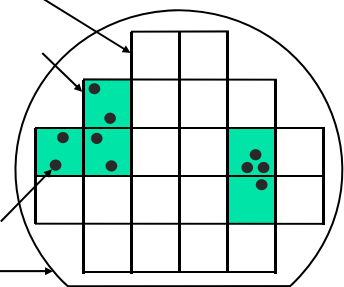
Clustered VLSI Defects

- Clustering which happens in reality gives higher yield



Good chips
Faulty chips
Defects
Wafer

Unclustered defects
Wafer yield = $12/22 = 0.55$



Good chips
Faulty chips
Defects
Wafer

Clustered defects (VLSI)
Wafer yield = $17/22 = 0.77$

Copyright 2011, M. Tahoori TDS I: Lecture 3 20



Clustering Effect

- Cluster THEOREM:

- The Boolean Quality Level achieved by a test
 - with Fault Coverage C,
 - for a process of Boolean Yield, Y

- $$QL = \frac{(1 - C)(1 - Y)e^{-(n-1)C}}{Y + (1 - C)(1 - Y)e^{-(n-1)C}}$$

- where n is average number of faults on a faulty die



What If (Motorola6802 Experiment)

- Single stuck-fault coverage were 96.6 % ?
 - Uniform Theory Predicts:
 - DL = 14,454 DPM or 174 Bad Die Pass Boolean Test
 - Cluster Theory with n=1 Predicts:
 - DL = 17,849 DPM or 217 Bad Die Pass Boolean Test
 - Cluster Theory with n=2 Predicts:
 - DL = 6,869 DPM or 84 Bad Die Pass Boolean Test
 - Experiment shows:
 - DL = 8,471 DPM or 103 Bad Die Pass Boolean Test



Yield Parameters

- Defect density (d)
 - Average number of defects per unit of chip area
- Chip area (A)
- Clustering parameter (α)
- Negative binomial distribution of defects,
 $p(x) = \text{Prob (number of defects on a chip} = x)$

$$= \frac{\Gamma(\alpha+x)}{x! \Gamma(\alpha)} \cdot \frac{(Ad/\alpha)^x}{(1+Ad/\alpha)^{\alpha+x}}$$

where Γ is the gamma function

$\alpha = 0$, $p(x)$ is a delta function (max. clustering)

$\alpha = \infty$, $p(x)$ is Poisson distr. (no clustering)

Copyright 2011, M. Tahoori

TDS I: Lecture 3

23



Yield Equation

$Y = \text{Prob (zero defect on a chip)} = p(0)$

$$Y = (1 + Ad/\alpha)^{-\alpha}$$

Example: $Ad = 1.0$, $\alpha = 0.5$, $Y = 0.58$

Unclassified defects: $\alpha = \infty$, $Y = e^{-Ad}$

Example: $Ad = 1.0$, $\alpha = \infty$, $Y = 0.37$

too pessimistic !

Copyright 2011, M. Tahoori

TDS I: Lecture 3

24



Modified Yield Equation

- Three parameters:
 - Fault density, f
 - Average number of stuck-at faults per unit chip area
 - Fault clustering parameter, β
 - Stuck-at fault coverage, T
- The modified yield equation:

$$Y(T) = (1 + T Af / \beta)^{-\beta}$$

Assuming that tests with 100% fault coverage ($T=1.0$) remove all faulty chips,

$$Y = Y(1) = (1 + Af / \beta)^{-\beta}$$

Copyright 2011, M. Tahoori

TDS I: Lecture 3

25



Defect Level

$$DL(T) = \frac{Y(T) - Y(1)}{Y(T)}$$

$$= 1 - \frac{(\beta + T Af)^{\beta}}{(\beta + Af)^{\beta}}$$

- T is the fault coverage of tests,
- Af is the average number of faults on the chip of area A
- β is the fault clustering parameter
- Af and β are determined by test data analysis.

Copyright 2011, M. Tahoori

TDS I: Lecture 3

26



Summary

- VLSI yield depends on two process parameters,
 - Defect density (d)
 - Clustering parameter (α)
- Yield drops as chip area increases
 - low yield means high cost
- Fault coverage measures the test quality
- Defect level (DL) or reject ratio is a measure of chip quality
- DL can be determined by an analysis of test data
- For high quality: $DL < 500 \text{ DPM}$,
 - Fault coverage should be $\sim 99\%$